

<http://goo.gl/Jv7Vj9>

**Course website**

KYOTO UNIVERSITY

*Statistical Machine Learning Theory*

## **Model Evaluation**

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## Topics:

# Performance measures and evaluation frameworks

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- You want to know the final performance of your model, or select the best one among possible models (or both)
- Performance measure: accuracy, precision/recall, DCG@k, AUC
- Evaluation framework: cross validation
- Model stacking

# Performance Measures

## Various performance measures:

Should be chosen according to your applications

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- There are various evaluation measure to quantify the performance of a trained model especially in supervised classification
  - Accuracy, precision/recall, DCG@ $k$ , AUC, ...
- They should be appropriately chosen depending on applications
  - Classification with decision thresholds: accuracy, precision/recall, ...
  - Classification without decision thresholds: AUC, ...
  - Ranking: DCG@ $k$ , ...

# Decision model and confusion matrix:

## Decisions on a dataset give a confusion matrix

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- The trained model gives confidence  $f(\mathbf{x})$  on given instance  $\mathbf{x}$  belonging to the positive class (+1)
- Assign +1 to  $\mathbf{x}$  whose  $f(\mathbf{x})$  is larger than decision threshold  $\tau$
- Fixing a model, a dataset, and a decision threshold gives a confusion matrix

		predicted label	
		positive	negative
true label	positive	#true positives 😊	#false negatives
	negative	#false positives	#true negatives 😊

# Accuracy, precision, recall, and F-measure: Basic predictive performance measures

- Accuracy: percentage of  $\frac{\text{\#true positives} + \text{\#true negatives}}{\text{\#all predictions}}$

- Precision/Recall

		predicted label	
		positive	negative
true label	positive	#true positives 😊	#false negatives
	negative	#false positives	#true negatives 😊

$$\text{-- Precision} = \frac{\text{\#true positives}}{\text{\#true positives} + \text{\#false positives}}$$

$$\text{-- Recall} = \frac{\text{\#true positives}}{\text{\#true positives} + \text{\#false negatives}}$$

$$\text{-- F-measure} = \frac{\text{Precision} \cdot \text{Recall}}{\text{Precision} + \text{Recall}}$$

- an integrated measure of precision and recall

# DCG@k:

## Performance measure for ranking

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- In ranking (of web pages), accuracy of top-ranked items is more important
- Precision@ $k$ : precision calculated using the top- $k$  scored items
- DCG(Discounted Cumulative Gain)@ $k$  is a weighted variant of Precision@ $k$ :  
$$\text{Precision@}k: \sum_{i=1}^k \frac{\text{rel}(i)}{\log(i+1)}$$
  - rel( $i$ ) is the relevance score for the  $i$ -th ranked item

# AUC:

## Performance measure not depending on the threshold

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- Evaluation needs fixing the decision threshold
- Imbalanced data generally results in a high accuracy
- AUC:
  - A performance measure directly defined with confidence score  $f(\mathbf{x})$
  - Probability of A being larger than B
    - A: confidence score of a randomly chosen positive instance
    - B: confidence score of a randomly chosen negative instance
  - takes 1 for perfect predictions, 0.5 for random predictions



# Evaluation Framework

# Evaluation framework:

## We want to predict model performance

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- Performance of a model for training data and that for future data are different
  - What we are interested in is the latter
- Many models have hyper-parameters to be specified by users
  - We want to tune them so that the final performance gets better

## First principle:

### Evaluation must use a dataset not used in training

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- You must not evaluate your classifier on the dataset you used for training
- Usually, first divide a given dataset into a training dataset and a test dataset
  1. Train a classifier using the training dataset
  2. Evaluate its performance on the test dataset
- Sometimes ordering of data instances (unintentionally) has some patterns in their labels
  - Partitioning should be done carefully

# Cross validation (for performance testing):

## A statistical framework for performance evaluation

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- You want to know the performance of the classifier (will be obtained using your algorithm) when it is deployed
- ( $K$ -fold) cross validation do this
- Divide a given dataset into  $K$  non-overlapping sets
  - Use  $K - 1$  of them for training
  - Use the remaining one for testing
- Changing the “test” datasets results in  $K$  measurements
  - Take their average to get a final performance estimate

# Cross validation for tuning hyper-parameters: A statistical framework for performance evaluation

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- Most of machine learning algorithms have hyper-parameters
  - Hyper-parameters: Parameters not automatically tuned in the training phase; given by users
- ( $K$ -fold) cross validation can be used for this
  - Use  $K - 1$  of  $K$  sets for training models for various hyper-parameter settings
  - Use the remaining one for testing
  - Choose the hyper-parameter setting with the best averaged performance
    - Note that this is **NOT** its final performance estimate

## Double loop of cross validation:

### Tuning hyper-parameters and performance evaluation

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- Sometimes you want to do both hyper-parameter tuning and performance evaluation
- Doing both with one  $K$ -fold cross validation is guilty
  - You see the test for tuning hyper-parameters
- Double loop cross validation
  - Outer loop for performance evaluation
  - Inner loop for hyper-parameter tuning
  - High computational costs...

# A simple alternative of double-loop cross validation: “Development set” approach

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- A simple alternative for the double-loop cross validation
- “Development set” approach
  - Use  $K - 2$  of  $K$  sets for training
  - Use one for tuning hyper-parameters
  - Use one for testing

# Model Stacking



# Model ensemble:

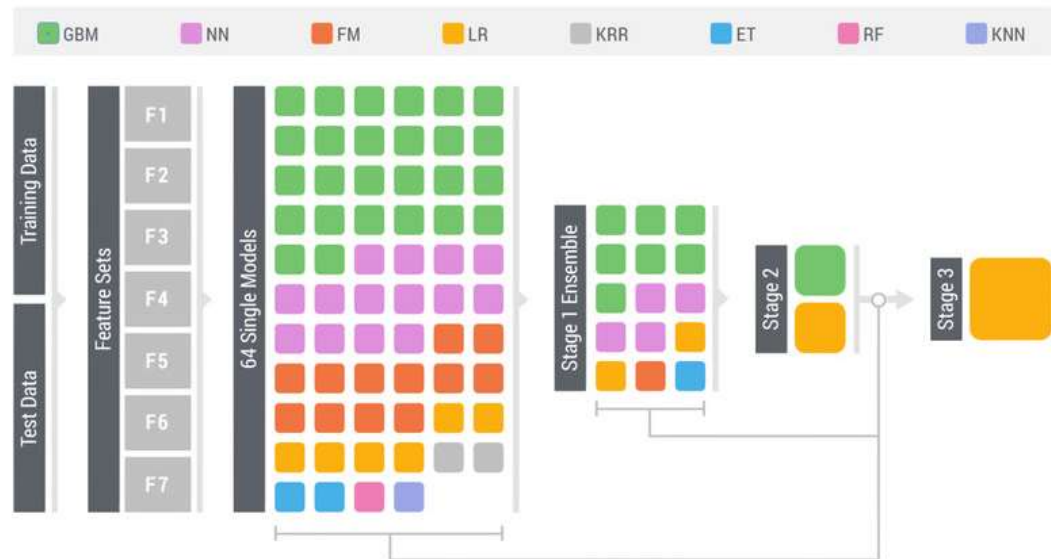
## Combines different models to improve performance

- One model cannot fit all
- Ensemble of different predictors to improve performance
- Commonly used technique in predictive modeling competitions (e.g. Kaggle)

### Three-Stage Ensemble



64 single + 15 ensemble + 2 ensemble + 1 blending



## Model stacking:

### An ensemble method to combine different models

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- Outputs of the level-0 models are inputs of the level-1 models
  - Original feature vector  $\mathbf{x}$
  - Outputs of the level-0 models  $\mathbf{y}$
  - New extended feature vector  $\tilde{\mathbf{x}} = \begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix}$  for level-1 models
- Stacking:
  - is similar to the multi-layer neural network
    - Stacked one-layer perceptron
  - but has heterogeneous components

# Difficulty in model stacking:

## An easy solution is biased

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- How can we train staked models?:
- An easy solution:
  1. train a classifier  $f$  using the training dataset  $L$
  2. add the prediction values of  $f$  as a new feature

.... seemingly works... but actually NOT
- Remember the first principle: you cannot make a prediction for the data you used in the training
  - The prediction value to the training data are biased because your model has been trained to reproduce the labels

## A solution:

### Use cross-validation to extend features

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- Divide a given dataset into  $K$  non-overlapping sets
  1. Use  $K - 1$  of them for training a model
  2. Use the model to add a new feature to the remaining set
  - Doing steps 1&2 for  $K$  holdout sets gives the new feature for the whole dataset
- Train the level-1 predictor using the extended dataset
- The level-0 predictor is (re-)trained using the original whole dataset
  - Because the extended feature for training the level-1 predictor is produced by different level-0 predictors